

# Minnesota State High School Mathematics League

## Individual Event

### 2003-04 Event 3A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. Give the coordinates of a point  $P$  that lies on the graphs of both of the lines

$$2x + 2y = -1$$

$$4x - 3y = 5$$

- \_\_\_\_\_ 2. The graphs of  $6x + 2y = 11$  and  $4x - 3y = -23$  pass through a common point  $Q$ . Write the equation of a line parallel to the  $x$ -axis that also passes through  $Q$ .

- \_\_\_\_\_ 3. Solve for  $x$  in terms of  $a$  and  $b$ , given that  $a \neq b$

$x =$  \_\_\_\_\_

$$ax + by = a$$

$$bx + ay = a$$

- \_\_\_\_\_ 4. There are many solutions to the set of three equations below. Express  $y$  and  $z$  in terms of  $x$  so that every real value of  $x$  gives a solution to these equations:

$y =$  \_\_\_\_\_

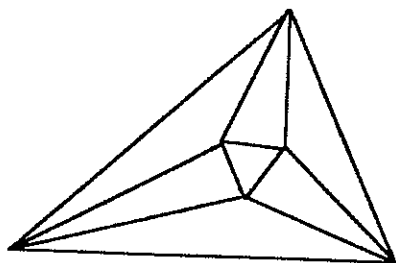
$z =$  \_\_\_\_\_

$$2x + y - z = -2$$

$$5x + 2y + z = 3$$

$$4x + y + 5z = 12$$

Name \_\_\_\_\_ Team \_\_\_\_\_



# Minnesota State High School Mathematics League

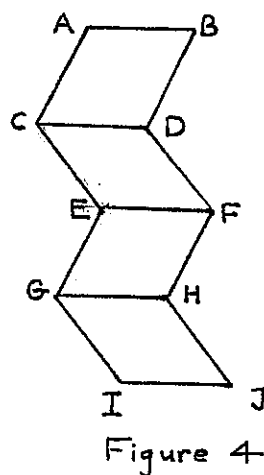
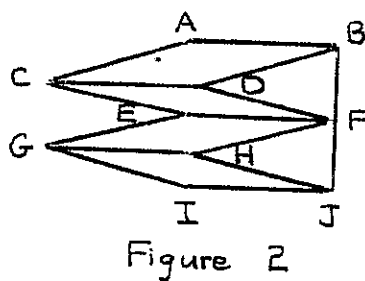
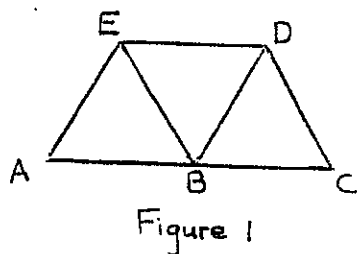
## Individual Event

### 2003-04 Event 3B

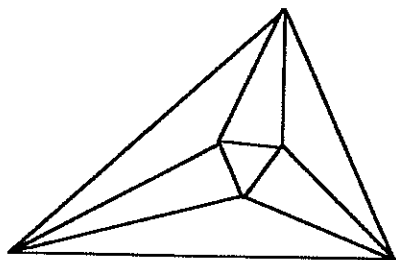
The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

Express all answers rounded correctly to three places to the right of the decimal.

- \_\_\_\_\_ 1. A child arranged seven match sticks of length 3 as indicated in Figure 1. What is the length of  $EC$ ?
- \_\_\_\_\_ 2. The same child went on to arrange on a table fourteen matchsticks of length 3 as shown in Figure 2, where  $AB \perp BJ$ , and  $BF = FJ$ . What is the length of  $BC$ ?
- \_\_\_\_\_ 3. Referring again to Figure 2, what is the area of parallelogram  $CGJF$ ?
- \_\_\_\_\_ 4. Suppose Figure 2 is changed slightly, deleting matchstick  $BJ$  and, while keeping the four rhombuses congruent, changing their interior angles as necessary so that  $C, E, H$ , and  $J$  are collinear (Figure 4). Then what will be the area of parallelogram  $CGJF$ ?



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# Minnesota State High School Mathematics League

## Individual Event

### 2003-04 Event 3C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

\_\_\_\_\_ 1.  $\triangle ABC$  has angles as indicated in Figure 1. The length of  $AB$  is 5. How long is  $AC$ ?

2. Express the square roots of the imaginary number  $i = \sqrt{-1}$  in the form  $a + bi$  where  $a$  and  $b$  are real.

\_\_\_\_\_ 3.  $\triangle ABC$  has sides  $AC = \sqrt{r}$ ,  $AB = \sqrt{3r}$ , and  $\angle BAC = 30^\circ$ . Find the measure of  $\angle ABC$ .

4. One method of solving

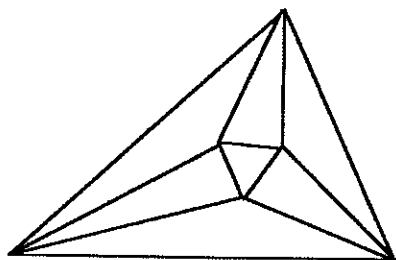
\_\_\_\_\_ (1)  $\sin x = \frac{1}{2} \sqrt{8 \cos x + 7}, \quad 0 \leq x \leq 360^\circ$

leads to an equation of the form

(2)  $a \cos^2 x + b \cos x + c = 0$

but (2) has a solution for  $x$  that is not a solution of (1). Find this extraneous solution.

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# Minnesota State High School Mathematics League

## Individual Event

### 2003-04 Event 3D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If  $\log_b M = 3$ , what is  $\log_M b$ ?

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2. If  $\log_4 A = r$ , what (in terms of  $r$ ) is  $\log_2 2A + \log_{16} 2A$ ?

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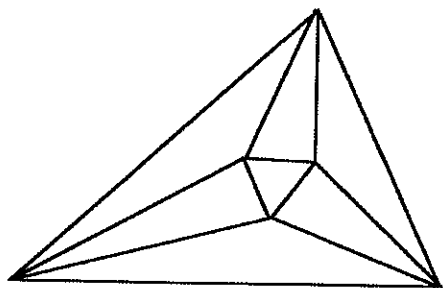
3. For what choices of  $b > 0$  will there be a solution to  $2\log_b x - \log_b(x^2 + 4) = 1$ ?

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4. Given that  $\log_3 2 = r$  and  $\log_3 b = s$ , express  $\log_b 72$  in terms of  $r$  and  $s$ .

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Name \_\_\_\_\_ Team \_\_\_\_\_



# Minnesota State High School Mathematics League

## Team Event

### 2003-04 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. A, B, and C entered the 5000 meter race. C won. At the moment that C crossed the finish line, B was 200 meters behind and A was 600 meters from the finish line. If B and A maintained the same pace as they had been running how many meters behind would A be when B crossed the finish line?

2.  $\triangle ABC$  has sides of  $AB = \sqrt{r}$ ,  $BC = 2\sqrt{r}$ , and  $CA = \sqrt{2r}$ . Find the measure in degrees of  $\angle ABC$  to the nearest tenth of a degree.

3. In Team Event 1, Problem 6, we found that if a point  $P$  is placed inside a rectangle  $ABCD$  so that  $AP = \sqrt{74}$ ,  $BP = \sqrt{34}$ , and  $CP = \sqrt{13}$ , then  $DP = \sqrt{53}$ . If the sides of the rectangle are integers, what must be its perimeter?

4. The innermost circle in Figure 4 has radius  $r$ . It is tangent to each of the eight circles of radius 1 that surround it, and each of these circles is tangent to its two small circle neighbors. What is the radius of the innermost circle?

5. Let  $0 < a < \frac{1}{2} < b < 1$ . Order the following, from smallest to largest:

$$\log_2 a, \log_{\frac{1}{2}} a, \log_2 b, \log_{\frac{1}{2}} b$$

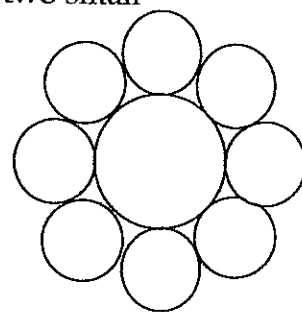


Figure 4

6. For  $k$  a positive integer and  $i = \sqrt{-1}$ , express  $\sum_{m=1}^{4k} mi^{m-1}$  in the form  $a(k) + b(k)i$  where  $a(k)$  and  $b(k)$  are real valued functions of  $k$ .

Team \_\_\_\_\_